

Homogenization of processes in nonlinear visco-elastic composites

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Abstract. The constitutive behaviour of a multiaxial visco-elastic material is here represented by the nonlinear relation

$$\varepsilon - A(x) : \int_0^t \sigma(x, \tau) d\tau \in \alpha(\sigma, x),$$

which generalizes the classical Maxwell model of visco-elasticity of fluid type. Here $\alpha(\cdot, x)$ is a (possibly multivalued) maximal monotone mapping, σ is the stress tensor, ε is the linearized strain tensor, and $A(x)$ is a positive-definite fourth-order tensor. The above inclusion is here coupled with the quasi-static force-balance law, $-\operatorname{div} \sigma = \vec{f}$. Existence and uniqueness of the weak solution are proved for a boundary-value problem.

Convergence to a two-scale problem is then derived for a composite material, in which the functions α and A periodically oscillate in space on a short length-scale. It is proved that the coarse-scale averages of stress and strain solve a single-scale homogenized problem, and that conversely any solution of this problem can be represented in that way. The homogenized constitutive relation is represented by the minimization of a time-integrated functional, and is rather different from the above constitutive law. These results are also retrieved via De Giorgi's notion of Γ -convergence. These conclusions are at variance with the outcome of so-called analogical models, that rest on an (apparently unjustified) mean-field-type hypothesis.

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